Non-Preemptive Scheduling with History-Dependent Execution Time

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Example of Task Set in the Old Model

n=3

T_1=50, D_1=10 T_2=150, D_2=15 T_3=500, D_3=500

C_1=4 C_2=7 C_3=5

Example of Task Set in the New Model

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n=3

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n=3

T_1=50, D_1=10 T_2=150, D_2=15 T_3=500, D_3=500

Example of Task Set in the New Model

n=3

T_1=50, D_1=10 T_2=150, D_2=15 T_3=500, D_3=500

Assume non-preemptive scheduling

Example of Task Set in the New Model

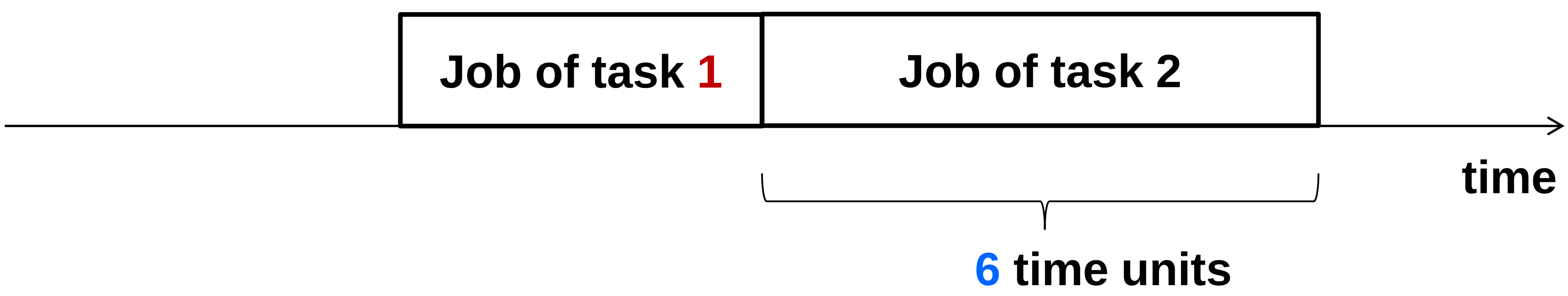
n=3

$T_1=50, D_1=10$

$T_2=150, D_2=15$

$T_3=500, D_3=500$

$ubc_2^2=6$, $lhubc_2^2=1$, $ihubc_2^{2,1}=1$



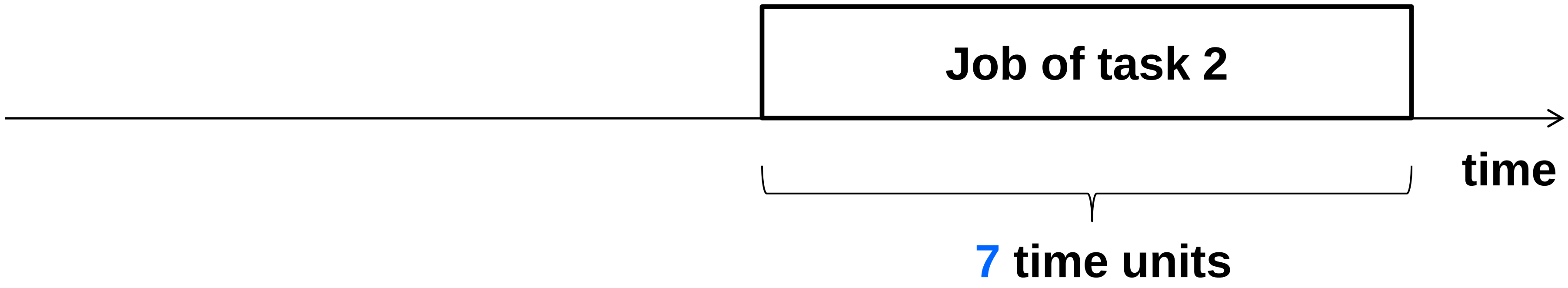
Execution time of a job when a pre-specified history is matched at run-time

Example of Task Set in the New Model

n=3

T_1=50, D_1=10 T_2=150, D_2=15 T_3=500, D_3=500

ubc_2^1=7, $lhubc_2^1$=0



Execution time of a job when the zero-length pre-specified history is matched at run-time

Example of Task Set in the New Model

n=3

T_1=50, D_1=10

T_2=150, D_2=15

T_3=500, D_3=500

$nhubc_2$=2

ubc_2^1=7, $lhubc_2^1$=0

ubc_2^2=6, $lhubc_2^2$=1, $ihubc_2^{2,1}$=1

Pre-specified histories of task τ_2 and their associated upper bounds on execution times

Example of Task Set in the New Model

$n=3$

$T_1=50, D_1=10$

$nhubc_1=2$

$ubc_1^1=4, lhubc_1^1=0$

$ubc_1^2=3, lhubc_1^2=1, ihubc_1^{2,1}=2$

$T_2=150, D_2=15$

$nhubc_2=2$

$ubc_2^1=7, lhubc_2^1=0$

$ubc_2^2=6, lhubc_2^2=1, ihubc_2^{2,1}=1$

$T_3=500, D_3=500$

$nhubc_3=1$

$ubc_3^1=5, lhubc_3^1=0$

Upper bounds on execution times of a job as a function of jobs that executed before it

Example of Task Set in the New Model

n=3

T_1=50, D_1=10
$nhlbc_1$=1
lbc_1^1=0, $lhlbc_1^1$=0

T_2=150, D_2=15
$nhlbc_2$=1
lbc_2^1=0, $lhlbc_2^1$=0

T_3=500, D_3=500
$nhlbc_3$=1
lbc_3^1=0, $lhlbc_3^1$=0

Lower bound on execution times of a job

Example of Task Set in the New Model

n=3

T_1=50, D_1=10
$nhlbc_1$=1
lbc_1^1=0, $lhlbc_1^1$=0
$nhubc_1$=2
ubc_1^1=4, $lhubc_1^1$=0
ubc_1^2=3, $lhubc_1^2$=1, $ihubc_1^{2,1}$=2

T_2=150, D_2=15
$nhlbc_2$=1
lbc_2^1=0, $lhlbc_2^1$=0
$nhubc_2$=2
ubc_2^1=7, $lhubc_2^1$=0
ubc_2^2=6, $lhubc_2^2$=1, $ihubc_2^{2,1}$=1

T_3=500, D_3=500
$nhlbc_3$=1
lbc_3^1=0, $lhlbc_3^1$=0
$nhubc_3$=1
ubc_3^1=5, $lhubc_3^1$=0

Upper and lower bounds on execution times of a job as a function of jobs that executed before it

Example of Task Set in the New Model

n=3

T_1=50, D_1=10

$nhlbc_1$=1

lbc_1^1=0, $lhlbc_1^1$=0

$nhubc_1$=2

ubc_1^1=4, $lhubc_1^1$=0

ubc_1^2=3, $lhubc_1^2$=1, $ihubc_1^{2,1}$=2

T_2=150, D_2=15

$nhlbc_2$=1

lbc_2^1=0, $lhlbc_2^1$=0

$nhubc_2$=2

ubc_2^1=7, $lhubc_2^1$=0

ubc_2^2=6, $lhubc_2^2$=1, $ihubc_2^{2,1}$=1

T_3=500, D_3=500

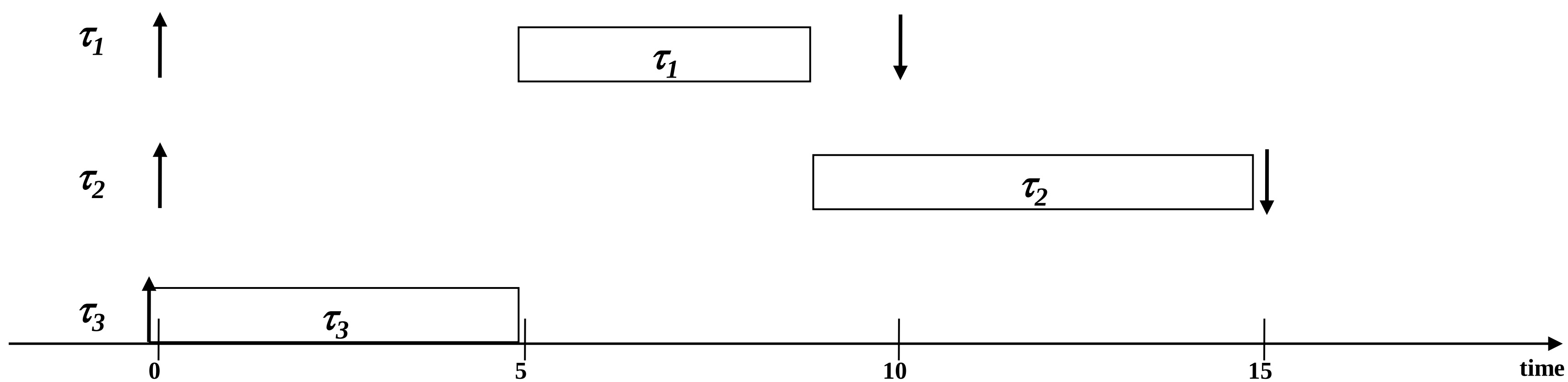
$nhlbc_3$=1

lbc_3^1=0, $lhlbc_3^1$=0

$nhubc_3$=1

ubc_3^1=5, $lhubc_3^1$=0

Schedule these tasks with non-preemptive fixed priority scheduling



This is the worst-case arrival for τ_2. With new model: deadline of τ_2 is met.

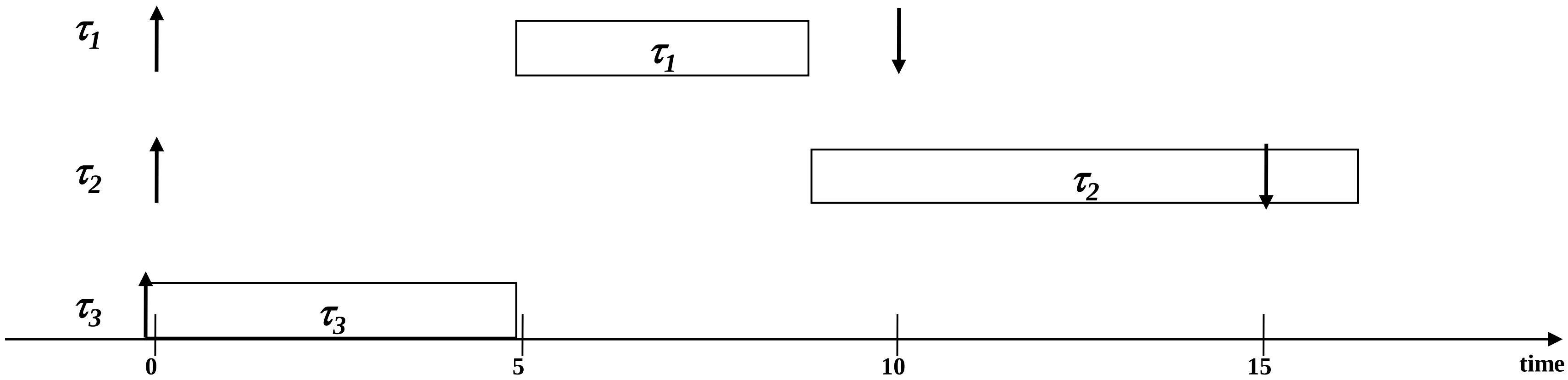
Example of Task Set in the Old Model

n=3

T_1=50, D_1=10 T_2=150, D_2=15 T_3=500, D_3=500

C_1=4 C_2=7 C_3=5

Schedule these tasks with non-preemptive fixed priority scheduling



This is the worst-case arrival for τ_2. With old model: deadline of τ_2 is missed.

Example of Task Set in the New Model

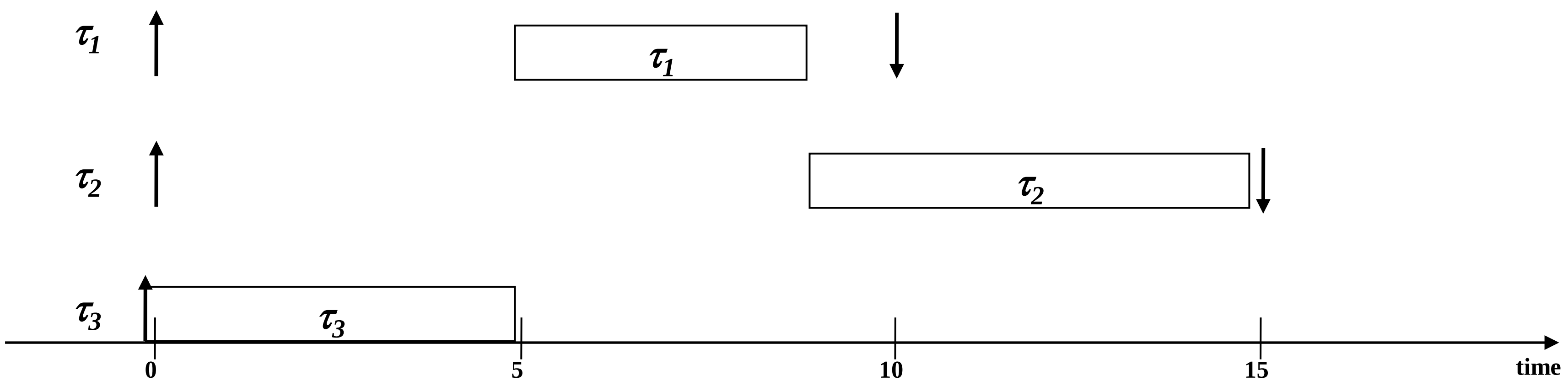
n=3

T_1=50, D_1=10
$nhlbc_1$=1
lbc_1^1=0, $lhlbc_1^1$=0
$nhubc_1$=2
ubc_1^1=4, $lhubc_1^1$=0
ubc_1^2=3, $lhubc_1^2$=1, $ihubc_1^{2,1}$=2

T_2=150, D_2=15
$nhlbc_2$=1
lbc_2^1=0, $lhlbc_2^1$=0
$nhubc_2$=2
ubc_2^1=7, $lhubc_2^1$=0
ubc_2^2=6, $lhubc_2^2$=1, $ihubc_2^{2,1}$=1

T_3=500, D_3=500
$nhlbc_3$=1
lbc_3^1=0, $lhlbc_3^1$=0
$nhubc_3$=1
ubc_3^1=5, $lhubc_3^1$=0

Schedule these tasks with non-preemptive fixed priority scheduling



This is the worst-case arrival for τ_2. With new model: deadline of τ_2 is met.

Example of Task Set in the New Model

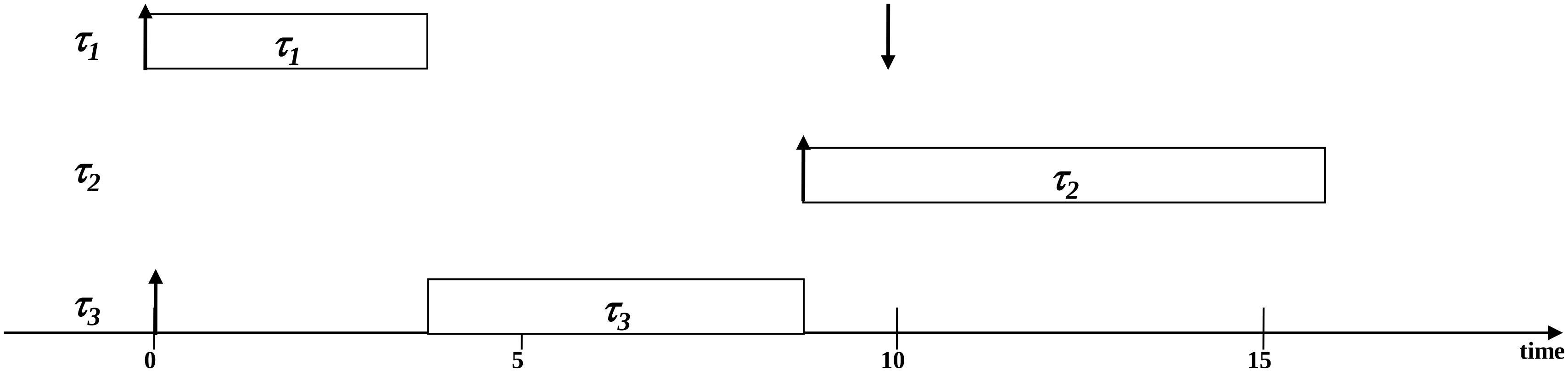
n=3

T_1=50, D_1=10
$nhlbc_1$=1
lbc_1^1=0, $lhlbc_1^1$=0
$nhubc_1$=2
ubc_1^1=4, $lhubc_1^1$=0
ubc_1^2=3, $lhubc_1^2$=1, $ihubc_1^{2,1}$=2

T_2=150, D_2=15
$nhlbc_2$=1
lbc_2^1=0, $lhlbc_2^1$=0
$nhubc_2$=2
ubc_2^1=7, $lhubc_2^1$=0
ubc_2^2=6, $lhubc_2^2$=1, $ihubc_2^{2,1}$=1

T_3=500, D_3=500
$nhlbc_3$=1
lbc_3^1=0, $lhlbc_3^1$=0
$nhubc_3$=1
ubc_3^1=5, $lhubc_3^1$=0

Schedule these tasks with non-preemptive fixed priority scheduling



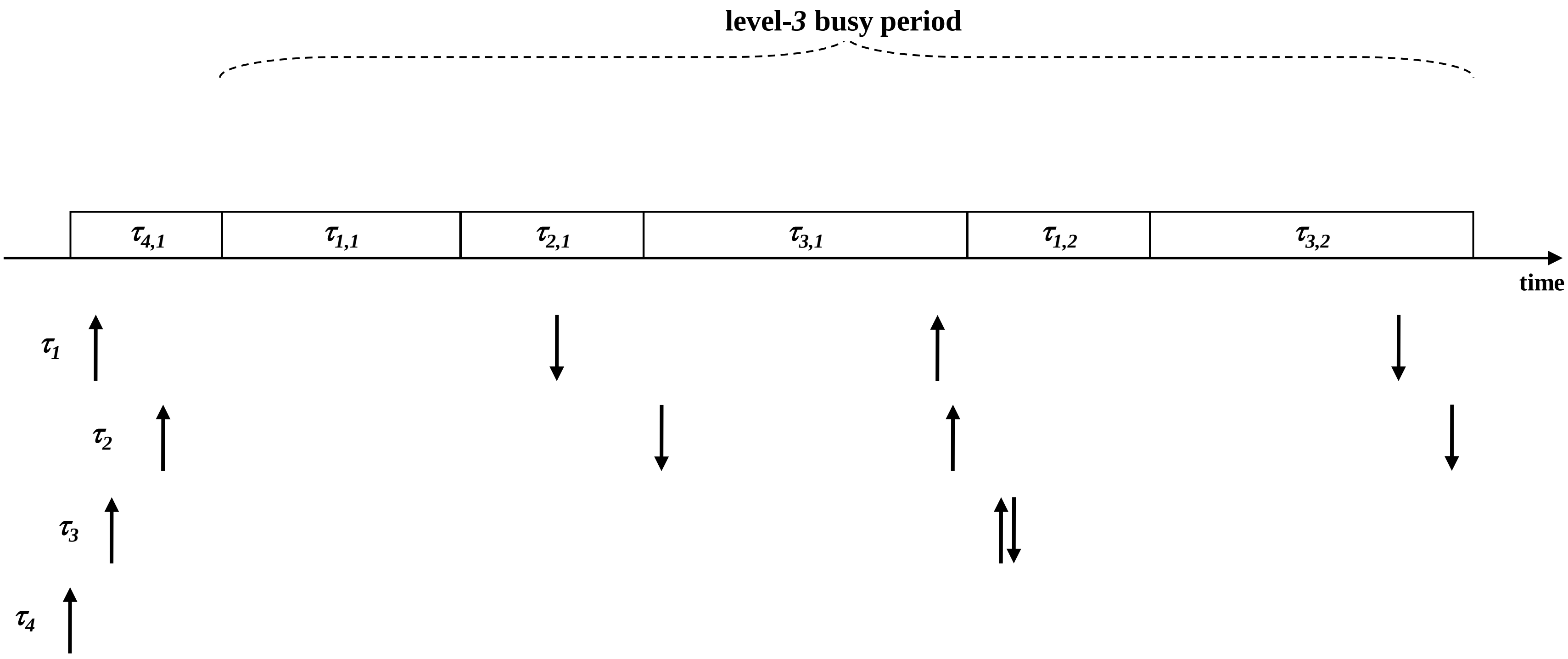
For another arrival for τ_2. Job of τ_2 has longer execution time but deadline of τ_2 is met.

Problem discussed in this presentation

How to perform schedulability analysis of tasks described with the new model under fixed-priority non-preemptive scheduling?

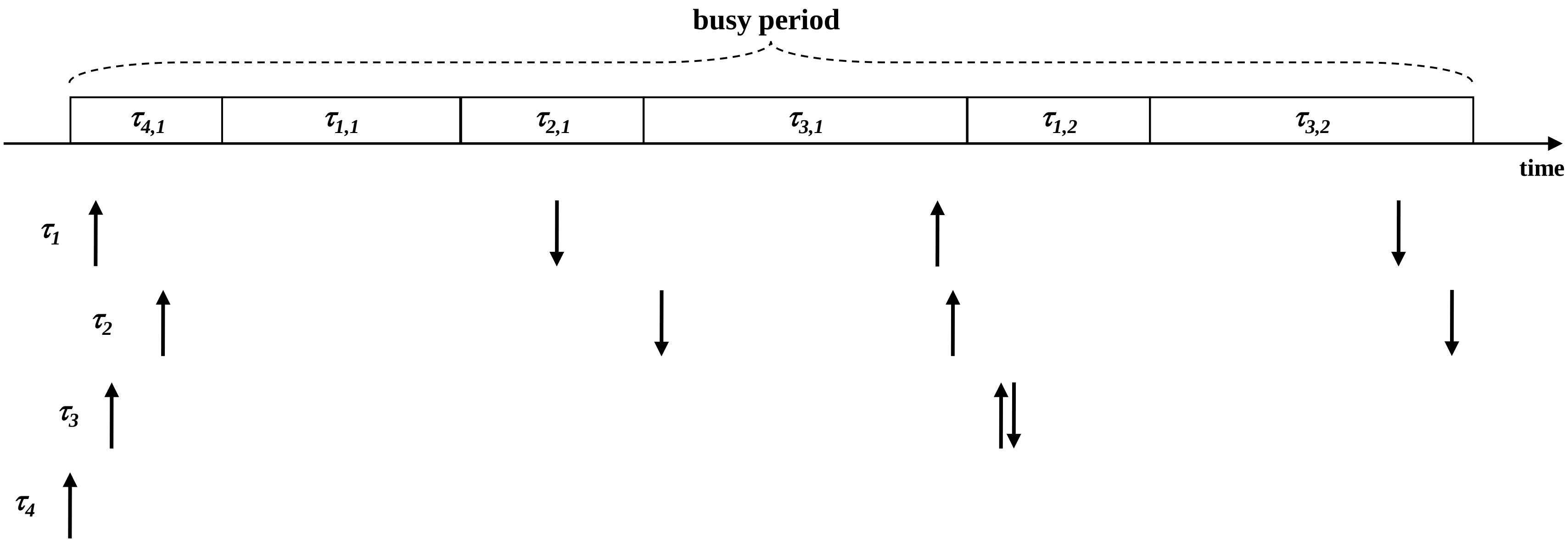
Ideas that do not work

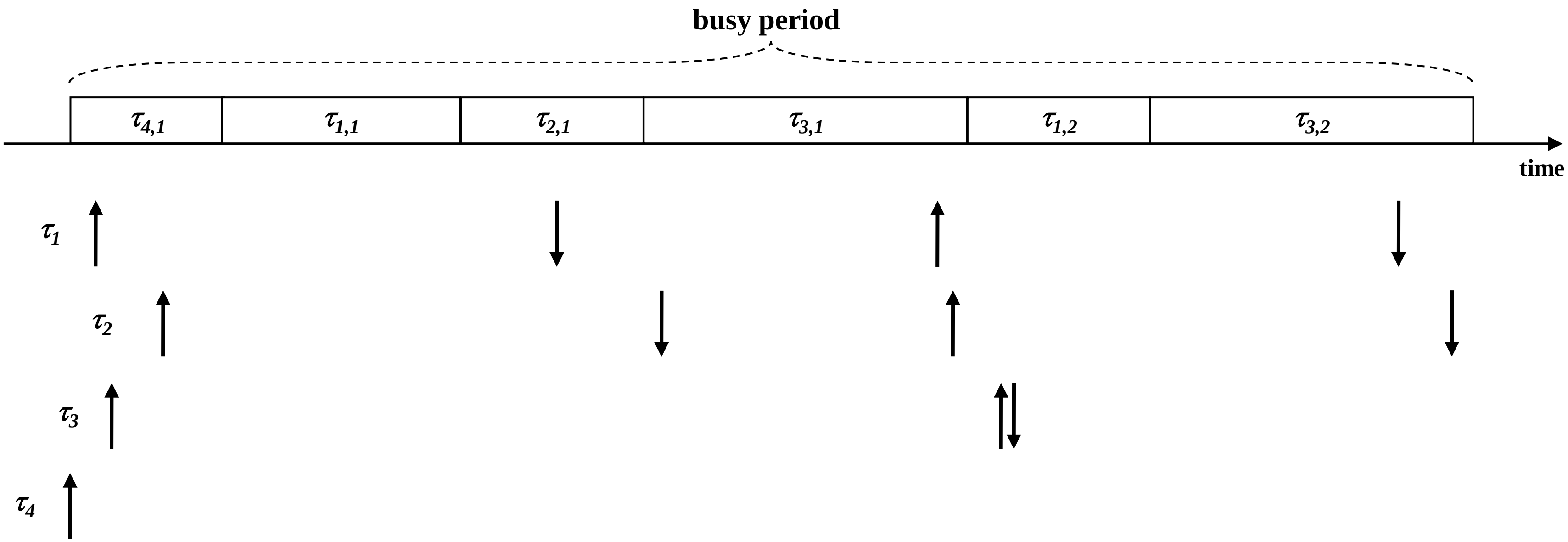
- L&L critical instant



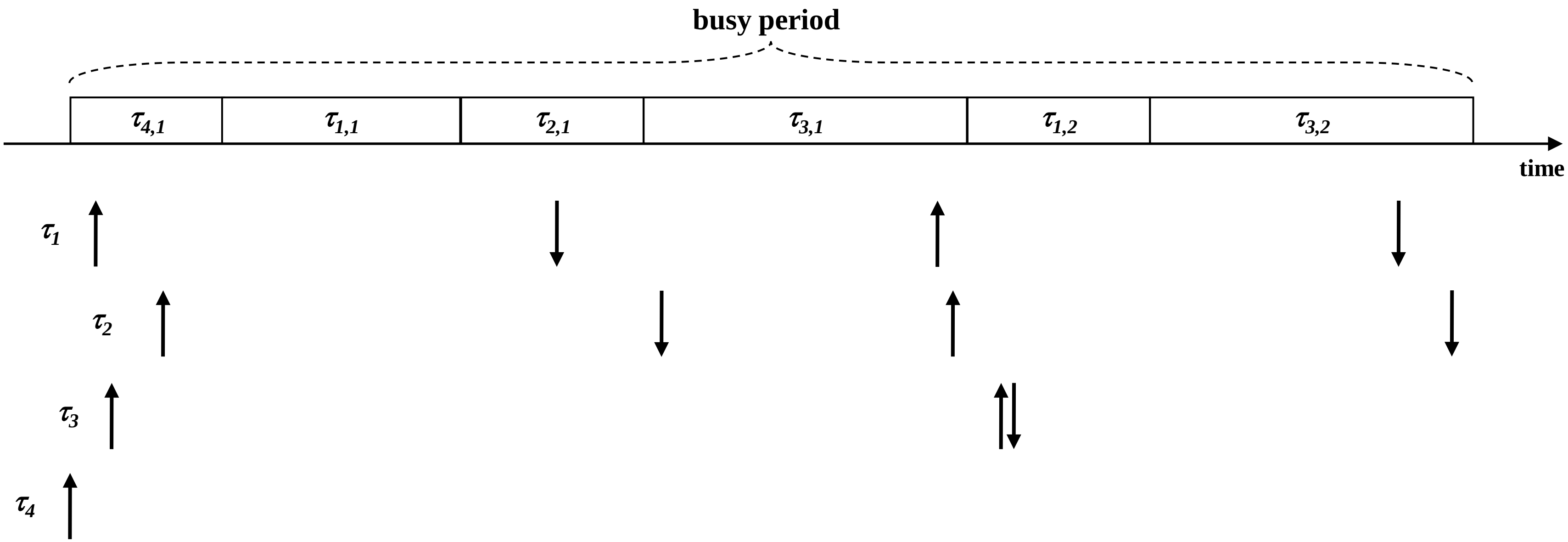
Ideas that do not work

- L&L critical instant
- Level-*i* busy period


busy period
$\tau_{4,1}$
$\tau_{1,1}$
$\tau_{2,1}$
$\tau_{3,1}$
$\tau_{1,2}$
$\tau_{3,2}$
time
τ_1
τ_2
τ_3
τ_4



1. L := Compute the maximum duration of a busy period
2. $Q_i := \lceil L/T_i \rceil$
3. **for** q := 1 to Q_i **do**
4. $R_{i,q}$:= compute the maximum response time of $\tau_{i,q}$ — the *q*:th job of task τ_i — in every busy period of length at most L
5. $R_i := \max_{q=1..Q} R_{i,q}$



An idea that almost works for computing the response time of task τ_i

1. L := Compute the maximum duration of a busy period
2. $Q_i := \lceil L/T_i \rceil$
3. **for** q := 1 to Q_i **do**
4. $R_{i,q}$:= compute the maximum response time of $\tau_{i,q}$ — the q:th job of task τ_i — in every busy period of length at most L
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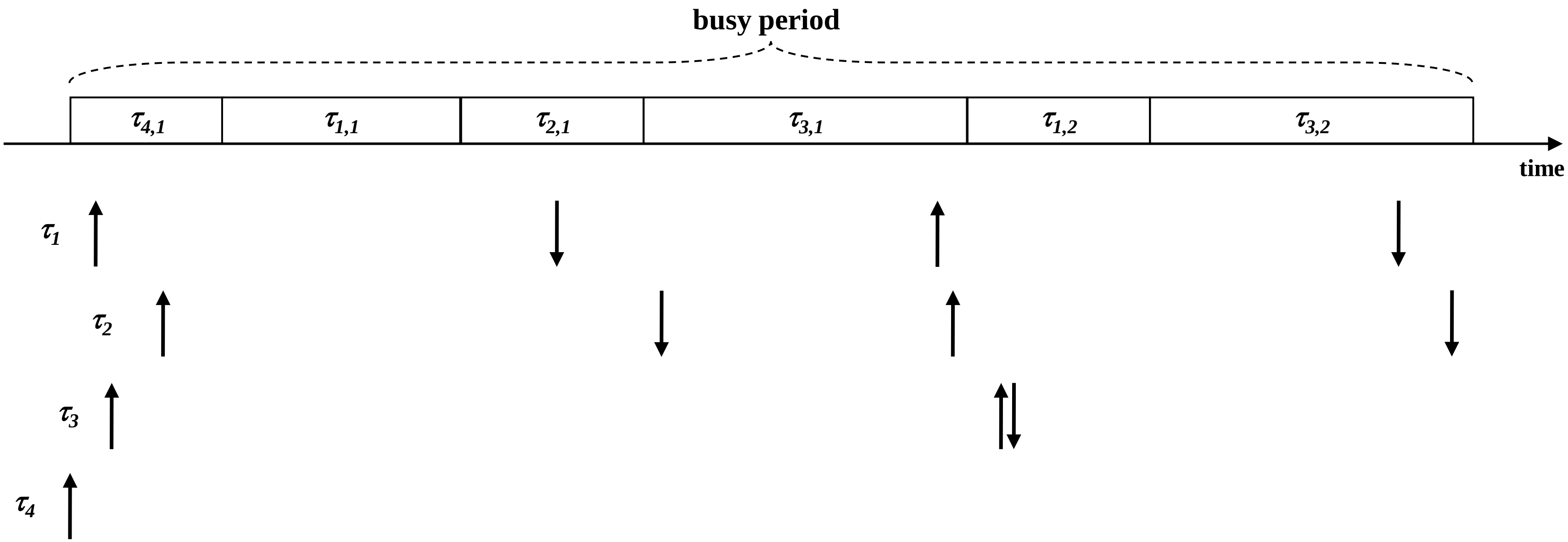
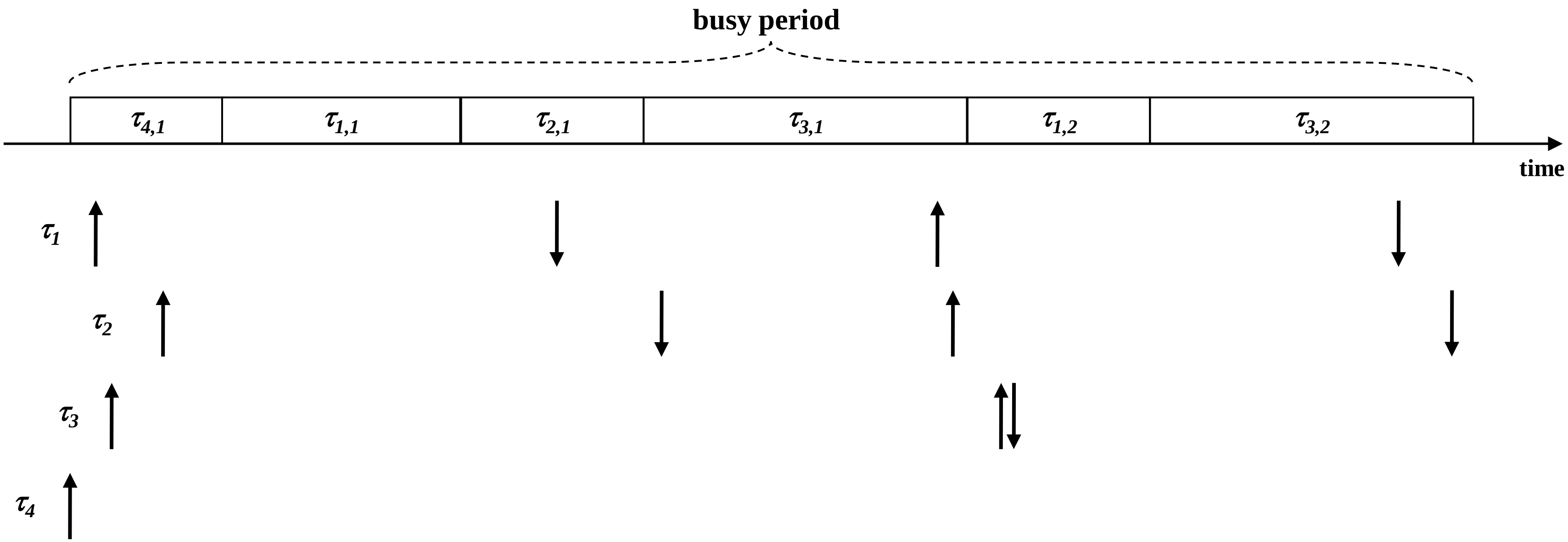


An idea that works for computing the response time of task τ_i

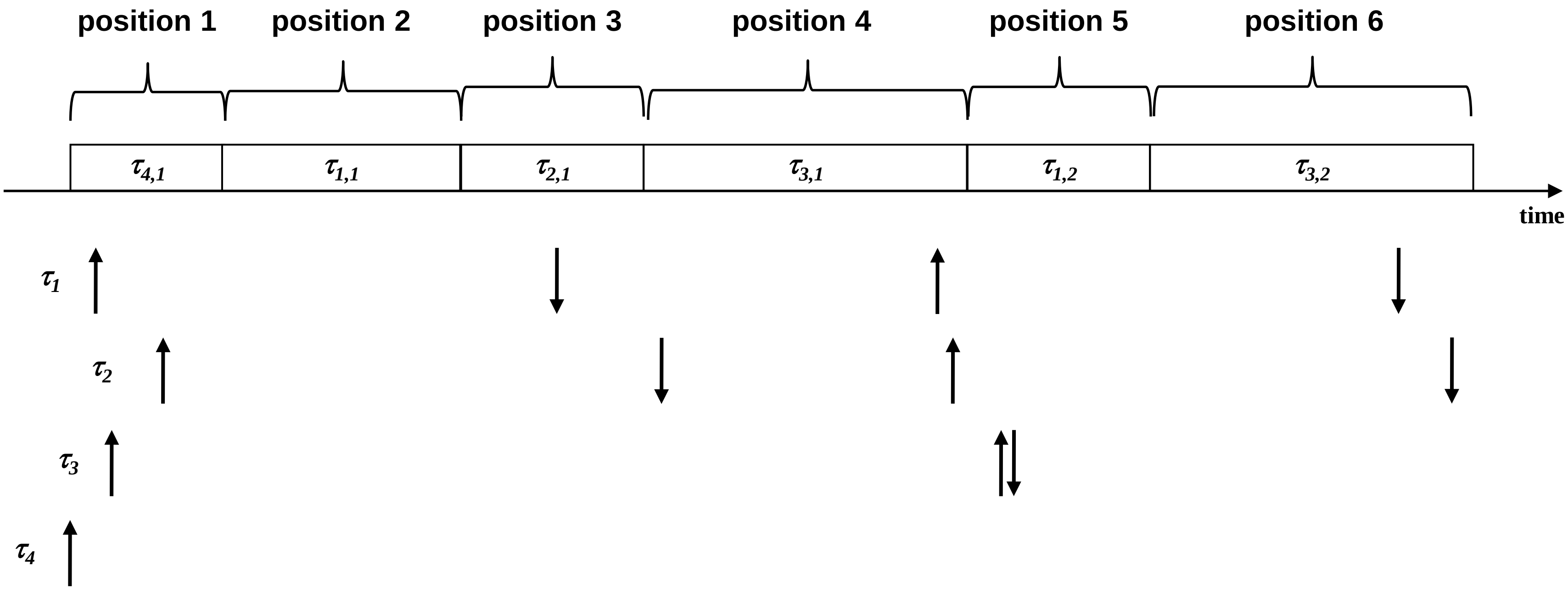
1. L := Compute the maximum duration of a busy period
2. $Q_i := \lceil L/T_i \rceil$
3. **for** q := 1 to Q_i **do**
4. <$R_{i,q}$,$valid_{i,q}$> := compute the maximum response time of $\tau_{i,q}$ — the q:th job of task τ_i — in every busy period of length at most L and compute whether $\tau_{i,q}$ exist in the busy period.
5. $\boldsymbol{R_i = max_{q=1..Q \text{ and } valid_{i,q}} R_{i,q}}$



<u>An idea that works for computing the response time of task</u>

1. L := Compute the maximum duration of a busy period
2. $Q_i := \lceil L/T_i \rceil$
3. **for** q := 1 to Q_i **do**
4. $\langle R_{i,q}, valid_{i,q} \rangle$:= compute the maximum response time of — the q:th job of task τ_i — in every busy period of length at most L and compute whether $\tau_{i,q}$ exist in the busy peri
5. $R_i = max_{q=1..Q \text{ and } valid_{i,q}} R_{i,q}$

How to perform step 1 and step 4?



An idea that works for computing the response time of task τ_i

1. L := Compute the maximum duration of a busy period
2. $Q_i := \lceil L/T_i \rceil$
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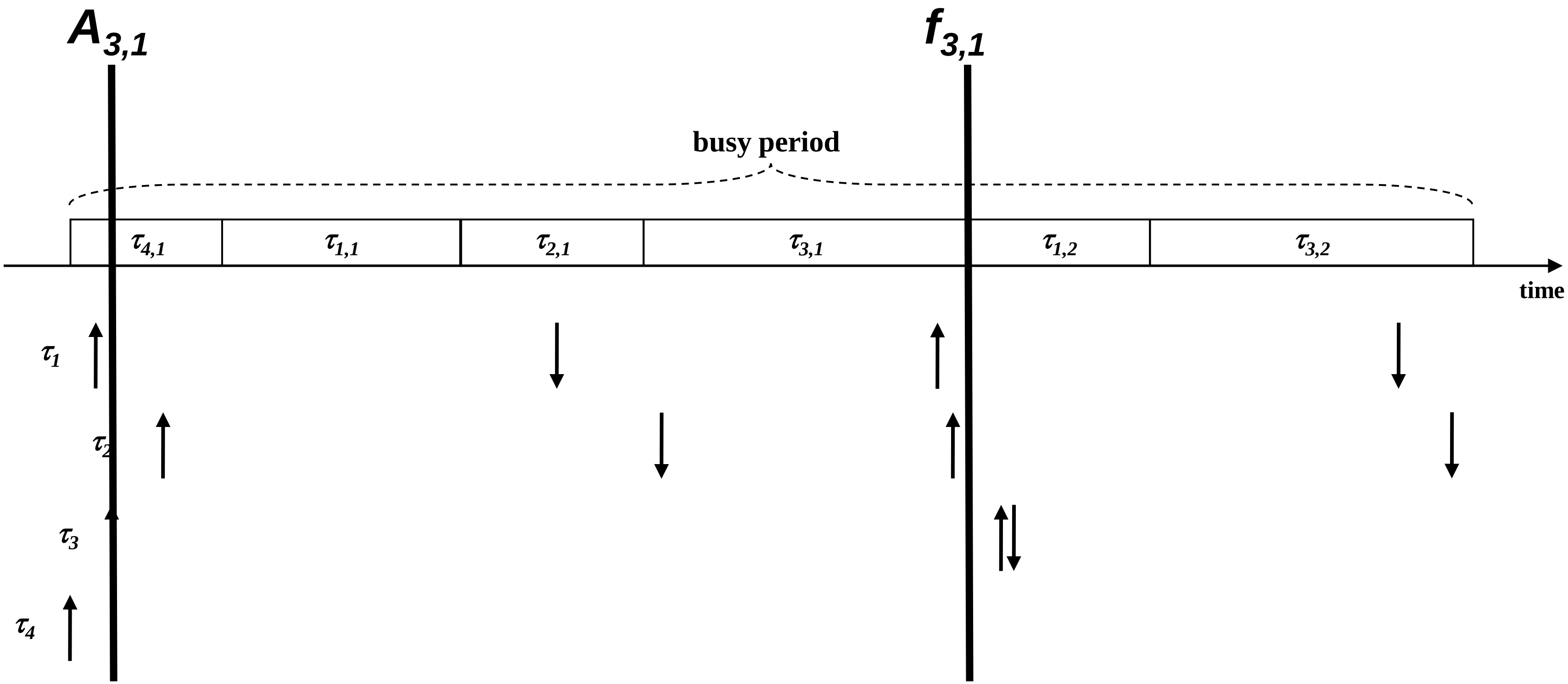

An idea that works for computing the response time of task τ_i

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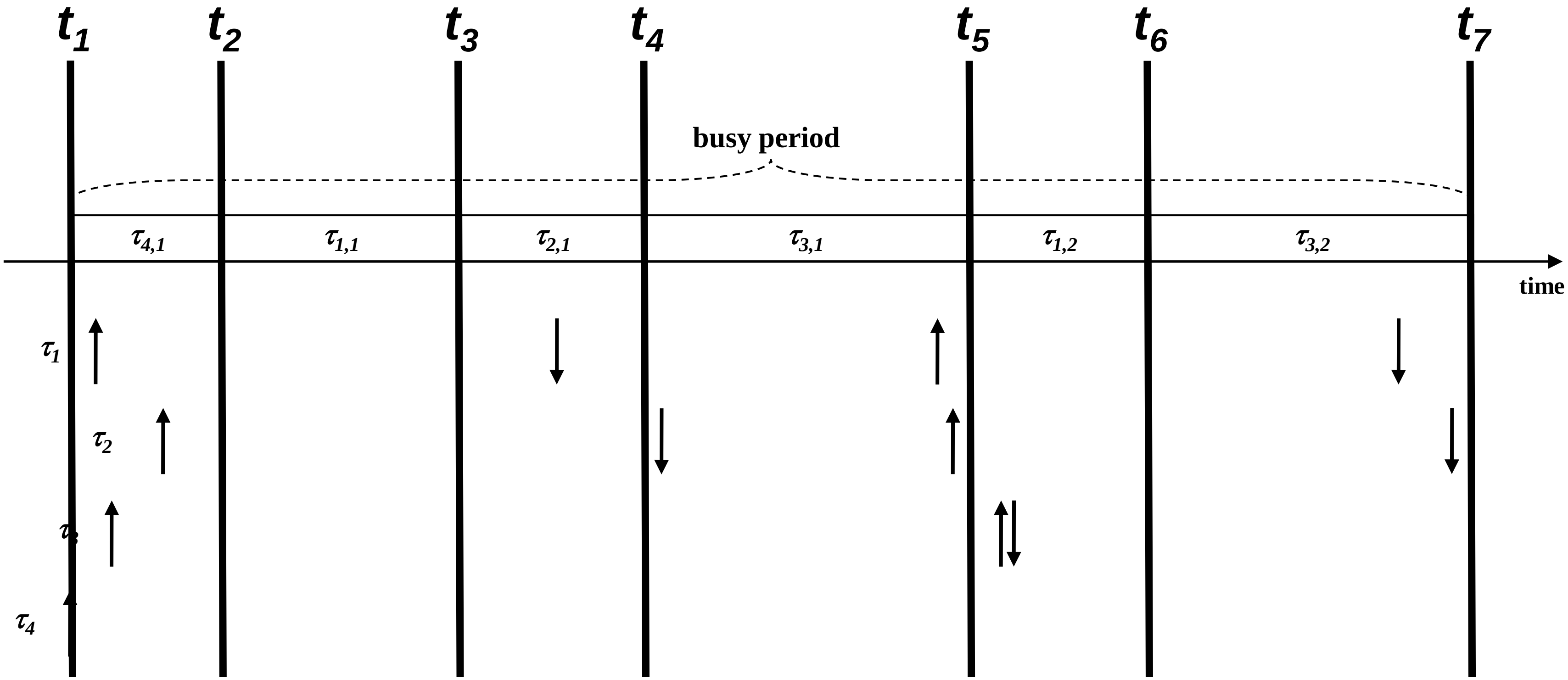

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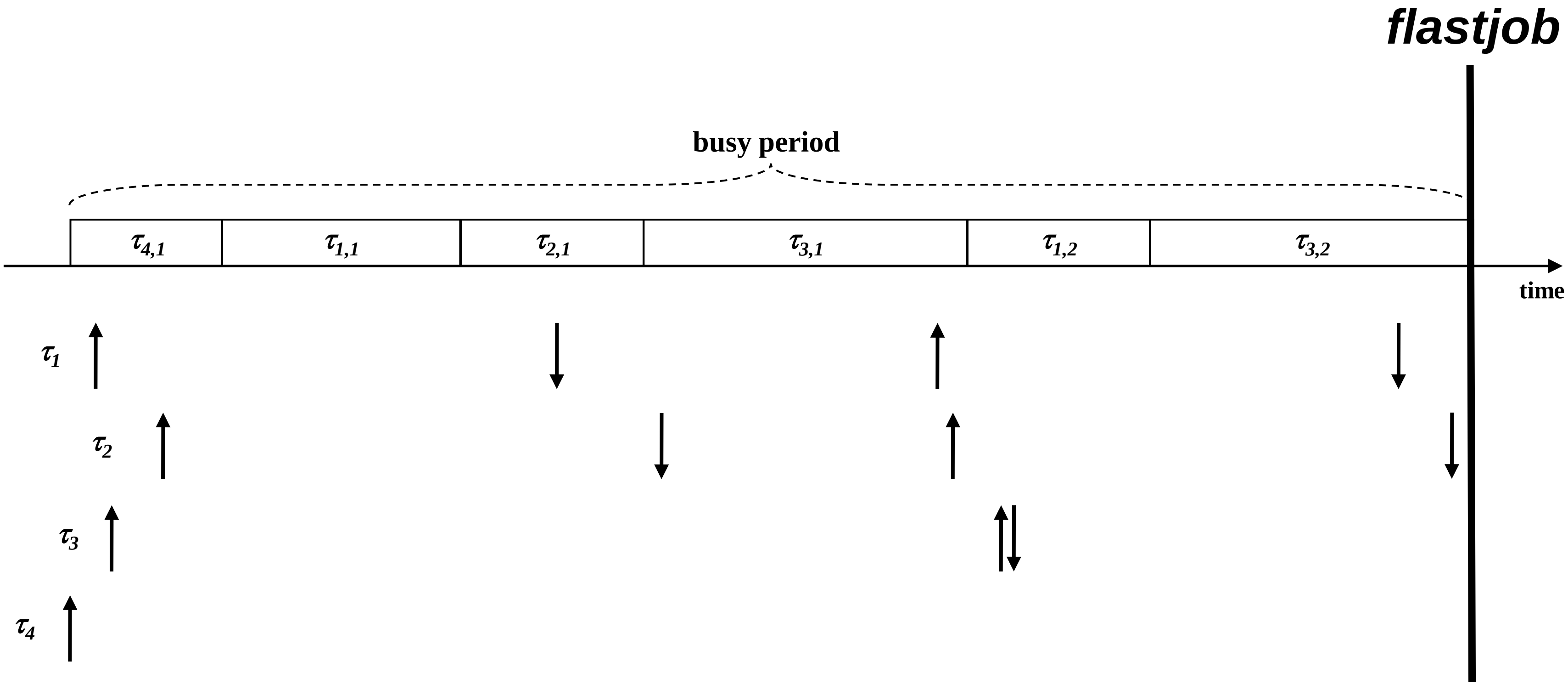

<u>An idea that works for computing the response time of task τ_i</u>

1. L := Compute the maximum duration of a busy period
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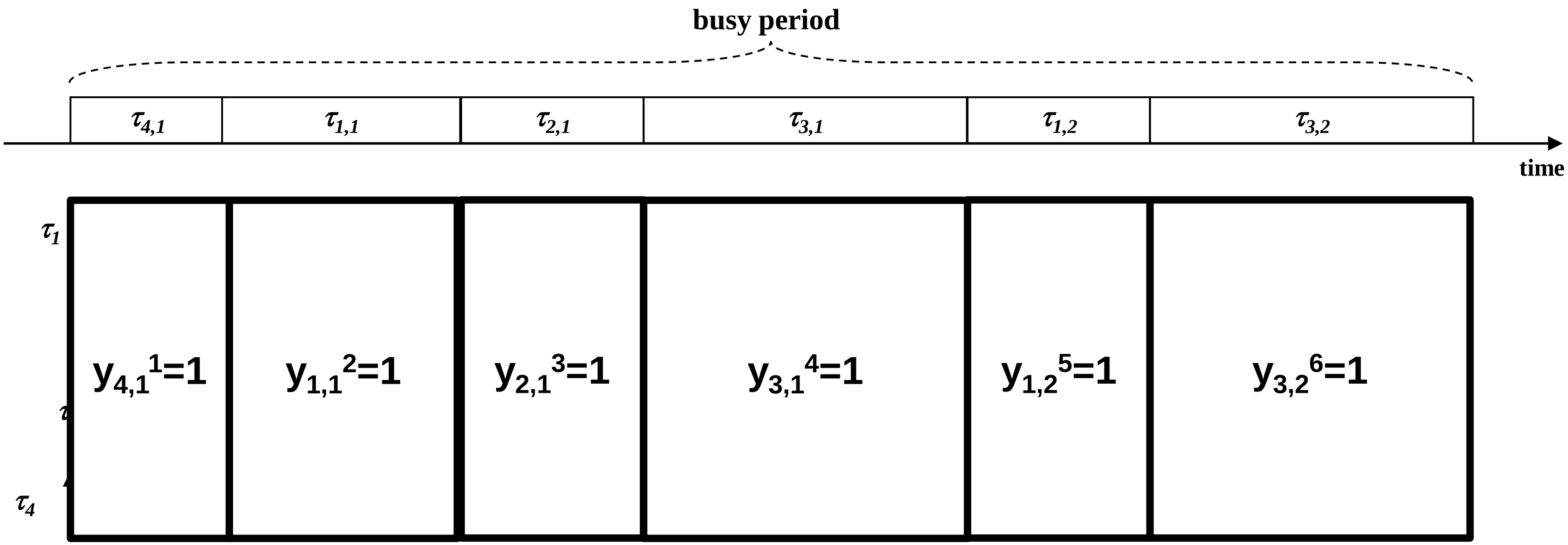

<u>An idea that works for computing the response time of task τ_i</u>

1. L := Compute the maximum duration of a busy period
2. $Q_i := \lceil L/T_i \rceil$
3. **for** q := 1 to Q_i **do**
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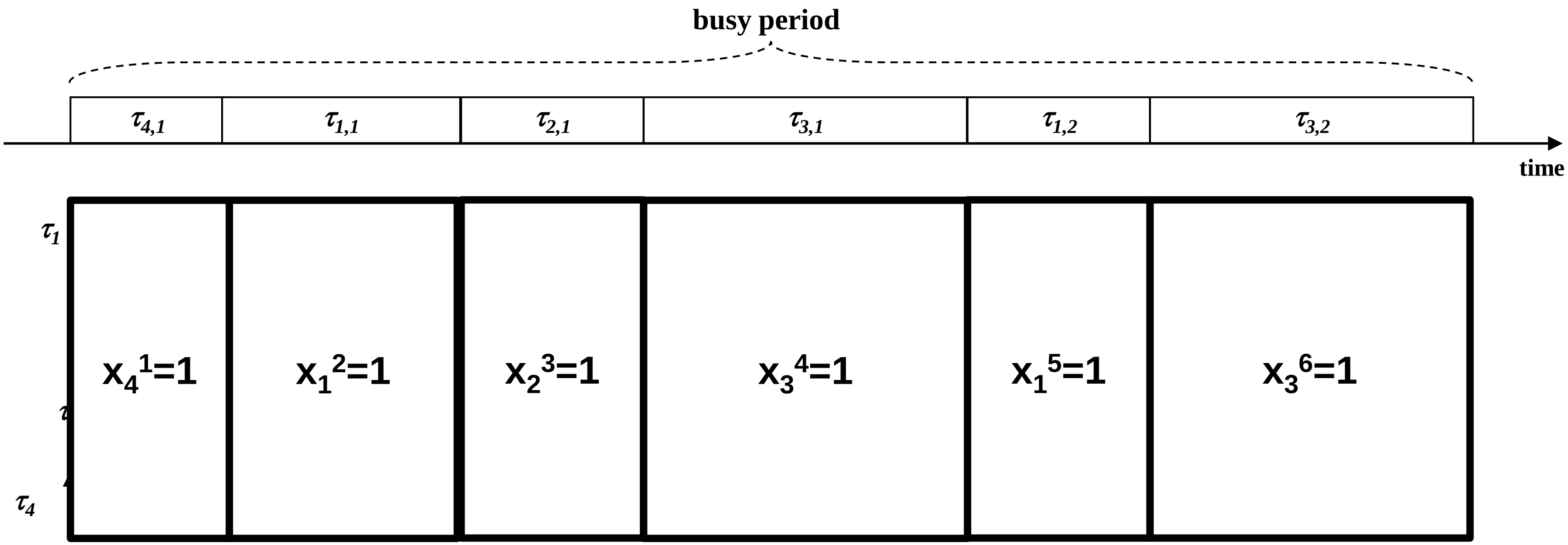
5. $\boldsymbol{R_i = max_{q=1..Q \text{ and } valid_{i,q}} R_{i,q}}$



An idea that works for computing the response time of task τ_i

1. L := Compute the maximum duration of a busy period
2. $Q_i := \lceil L/T_i \rceil$
3. **for** q := 1 to Q_i **do**
4. $\langle R_{i,q}, valid_{i,q} \rangle$:= compute the maximum response time of $\tau_{i,q}$ — the q:th job of task τ_i — in every busy period of length at most L and compute whether $\tau_{i,q}$ exist in the busy period.
5. $\boldsymbol{R_i = max_{q=1..Q \; and \; valid_{i,q}} R_{i,q}}$

Perform step 1: Compute the maximum duration of a busy period

Represent a schedule.

x_j^p = 1 iff a job of task τ_j executes in position p in busy period

$y_{j,k}^p$ = 1 iff job $\tau_{j,k}$ executes in position p in busy period

t_k = time of k:th context switch in busy period

Other variables

$A_{j,k}$ = arrival time of $\tau_{j,k}$

$f_{j,k}$ = finishing time of $\tau_{j,k}$

ftlastjob= time when the busy period ends

Maximize *ftlastjob*- t_1 subject to constraints …. [see paper]

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1. L := Compute the maximum duration of a busy period
2. $Q_i := \lceil L/T_i \rceil$
3. **for** q := 1 to Q_i **do**
4. <$R_{i,q}$,$valid_{i,q}$> := compute the maximum response time of $\tau_{i,q}$ — the q:th job of task τ_i — in every busy period of length at most L and compute whether $\tau_{i,q}$ exist in the busy period.
5. $\boldsymbol{R_i = max_{q=1..Q \text{ and } valid_{i,q}} R_{i,q}}$

Perform step 4: Compute the maximum response time of $\tau_{i,q}$ during every busy period of duration at most *L*

Represent a schedule.

x_j^p = 1 iff a job of task τ_j executes in position *p* in busy period

$y_{j,k}^p$ = 1 iff job $\tau_{j,k}$ executes in position *p* in busy period

t_k = time of *k*:th context switch in busy period

Other variables

$A_{j,k}$ = arrival time of $\tau_{j,k}$

$f_{j,k}$ = finishing time of $\tau_{j,k}$

ftlastjob= time when the busy period ends

Maximize $f_{j,k} - A_{j,k}$ subject to constraints …. [see paper]

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1. *L* := Compute the maximum duration of a busy period
2. $Q_i := \lceil L/T_i \rceil$
3. **for** q := 1 to Q_i **do**
4. <$R_{i,q}$,$valid_{i,q}$> := compute the maximum response time of $\tau_{i,q}$ — the *q*:th job of task τ_i — in every busy period of length at most *L* and compute whether $\tau_{i,q}$ exist in the busy period.
5. $\boldsymbol{R_i = max_{q=1..Q \text{ and } valid_{i,q}} R_{i,q}}$

Conclusion

It is possible to compute exact response times of tasks scheduled by non-preemptive fixed-priority scheduling where execution times depend on history.

Thanks for listening!